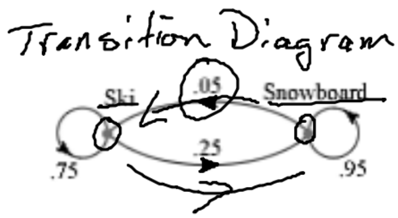
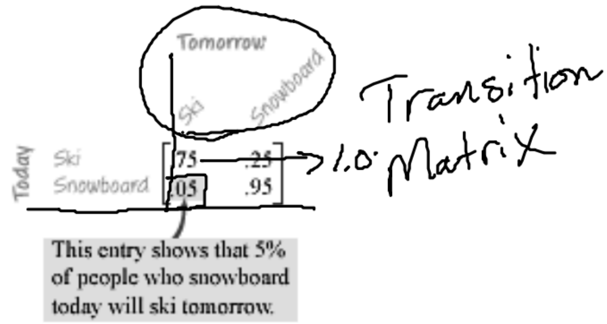
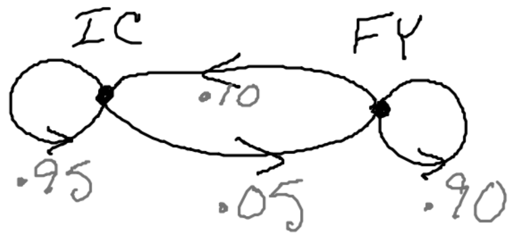
6.1 Matrix Representations





Inv.



	IC	FY
W1	220	20
W2	211	29
W3	203	37

	IC	FY	
IC	.95	.05	1.0
FY	.10	.90	1.0

$$\begin{bmatrix} 209 & 11 \\ 2 & 18 \end{bmatrix}$$

211 29

$$\begin{bmatrix} 200 & 11 \\ 3 & 26 \end{bmatrix}$$

Dimensions

Rows ⟶

$$\begin{bmatrix} 200 & 11 \\ 3 & 26 \end{bmatrix}$$

columns ↓

Rows × Columns

[2 × 2]

[A]

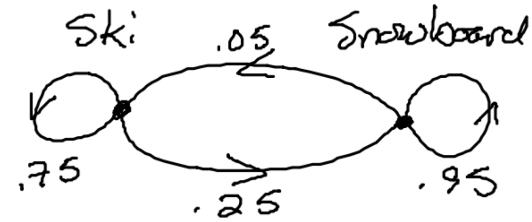
Ex. A

	A	B	C	D
x	1	2	-3	-2
y	2	-2	-1	1

[2×4]

Ex. B

4/11/12



Skiers 260

Snowboarders 40

$$\begin{bmatrix} 195 & 65 \\ 2 & 38 \end{bmatrix}$$

$$\begin{bmatrix} 197 & 103 \end{bmatrix}$$

$$\begin{bmatrix} 148 & 49 \\ 5 & 98 \end{bmatrix}$$

$$\begin{bmatrix} 153 & 147 \end{bmatrix}$$

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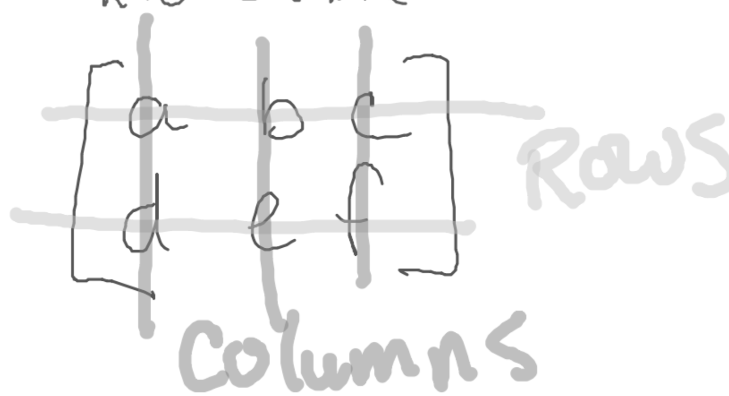
M_{32} $M_{3,2}$

↑ row ↑ column



6.2 Matrix Operations p. 307 4/13/12

3×2 3×2 2×2

$$[A] = \begin{bmatrix} 83 & 33 \\ 65 & 20 \\ 98 & 50 \end{bmatrix} \qquad [B] = \begin{bmatrix} 80 & 25 \\ 65 & 15 \\ 105 & 55 \end{bmatrix} \qquad [C] = \begin{bmatrix} 27 & 25 \\ 19 & 3 \end{bmatrix}$$

$[A]+[C]$ = can't be done

$$[A]+[B] = \begin{bmatrix} 163 & 58 \\ 130 & 35 \\ 203 & 105 \end{bmatrix} \quad 3\times 2$$

$$[A]-[B] = \begin{bmatrix} 3 & 8 \\ 0 & 5 \\ -7 & -5 \end{bmatrix} \quad 3\times 2$$

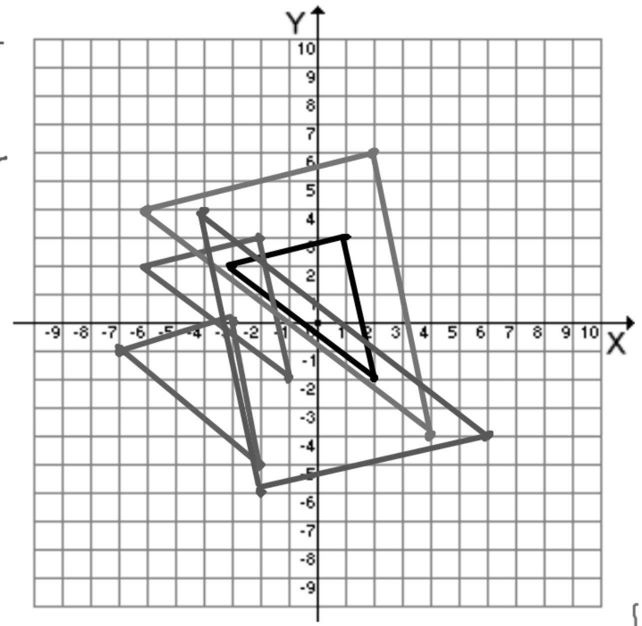
Ex. A $\overset{-}{\leftarrow} X \overset{+}{\rightarrow}$ **a)** $\begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} \begin{matrix} x \\ y \end{matrix}$ $\uparrow +$ y $\downarrow -$

translated Left 3 $\begin{bmatrix} -6 & -2 & -1 \\ 2 & 3 & -2 \end{bmatrix}$

$x - 3$

b) $\begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} + \begin{bmatrix} -4 & -4 & -4 \\ -3 & -3 & -3 \end{bmatrix}$

translate left 4, down 3



c) $\underline{\;2\;} \cdot \begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -6 & 2 & 4 \\ 4 & 6 & -4 \end{bmatrix}$

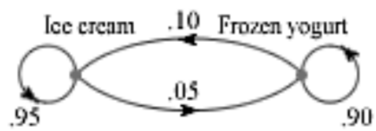
scalar multiplication

dilation

$\frac{1}{2} \cdot$ Shrink scalar

$\begin{bmatrix} 6 & -2 & -4 \\ -4 & -6 & 4 \end{bmatrix}$ - vertical + horizontal flip

Ex. B



Ice cream = 220
Frozen yogurt = 20

$1 \times 2 \qquad 2 \times 2$

$$\begin{bmatrix} 220 & 20 \end{bmatrix} \times \begin{bmatrix} .95 & .05 \\ .10 & .90 \end{bmatrix} = \begin{bmatrix} 211 & 29 \end{bmatrix}$$

$2 \times 2 \qquad 1 \times 2$

$(220 \times .95) + (20 \times .10) =$ ____

$1 \times 3 \qquad 2 \times 3$

$$\begin{bmatrix} 1 & 5 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 7 & 3 \\ 4 & 9 & 6 \end{bmatrix} =$$ Can't be done

Ex. C

$$\underset{2\times 2}{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix}} = \underset{2\times 3}{\begin{bmatrix} 3 & -1 & -2 \\ 2 & 3 & -2 \end{bmatrix}}$$

Matrix Operations

Addition

dimensions must be the same

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Scalar Multiplication

$$a\begin{bmatrix} b & c & d \\ e & f & g \end{bmatrix} = \begin{bmatrix} a \cdot b & a \cdot c & a \cdot d \\ a \cdot e & a \cdot f & a \cdot g \end{bmatrix}$$

$n_{1,1}$ = Matrix n row 1 column 1

Matrix multiplication

middle dimensions must be the same

$a \times b \quad b \times c$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f & g \\ h & k & m \end{bmatrix} = \begin{bmatrix} ae+bh & af+bk & ag+bm \\ ce+dh & cf+dk & cg+dm \end{bmatrix}$$

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#1-4

$2x + y = 5$
$5x + 3y = 13$

$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$ $\left[\begin{array}{cc|c} 2 & 1 & 5 \\ 5 & 3 & 13 \end{array}\right]$

Augmented Matrix

$\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$ reduced-row echelon form (rref)

Can trade one row with another

$\left[\begin{array}{cc|c} 2 & 1 & 5 \\ 5 & 3 & 13 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 5 & 3 & 13 \\ 2 & 1 & 5 \end{array}\right]$

Can multiply a row by any number

$$\left[\begin{array}{cc|c} 2 & 1 & 5 \\ 5 & 3 & 13 \end{array}\right] \quad R_1 \times 3 \rightarrow R_1 \quad \left[\begin{array}{cc|c} 6 & 3 & 15 \\ 5 & 3 & 13 \end{array}\right]$$

Can add or subtract one row from another row

$$\left[\begin{array}{cc|c} 6 & 3 & 15 \\ 5 & 3 & 13 \end{array}\right] \quad R_1 - R_2 \rightarrow R_1 \quad \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 5 & 3 & 13 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 5 & 3 & 13 \end{array}\right] \quad R_2 - 5R_1 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 3 & 3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 3 & 3 \end{array}\right] \xrightarrow{R_2 \div 3 \to R_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

$(2, 1)$

- Multiply a row by a number
- trade a row with another row
- add or subtract one row from/with another row

Ex. B

a	b	c
\$14.00	\$18.50	\$23.25

$$14a + 18.50b + 23.25c = \$909.00$$

$$b - c = 22$$

$$a + b + c = 50$$

$$\left[\begin{array}{ccc|c} 14 & 18.50 & 23.25 & 909 \\ 0 & 1 & -1 & 22 \\ 1 & 1 & 1 & 50 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 14 & 18.50 & 23.25 & 909 \\ 0 & 1 & -1 & 22 \\ 1 & 1 & 1 & 50 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right]$$

$R_1 - 14R_3 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 0 & 4.50 & 9.25 & 209 \\ 0 & 1 & -1 & 22 \\ 1 & 1 & 1 & 50 \end{array}\right]$$

$R_1 - 4.50R_2 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 0 & 0 & 13.75 & 110 \\ 0 & 1 & -1 & 22 \\ 1 & 1 & 1 & 50 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 1 & -1 & 22 \\ 0 & 0 & 13.75 & 110 \end{array}\right]$$

$R_3 \times \frac{1}{13.75} \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 1 & -1 & 22 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 1 & -1 & 22 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

$R_1 - R_2 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 28 \\ 0 & 1 & -1 & 22 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

$R_2 + R_3 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 28 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

$R_1 - 2R_3 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

$(12, 30, 8)$

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#1-6

6.4 Solving Systems of Inverse Matrices p. 327 4/23/12

Multiplicative identity - $\times 1$ same answer out as you put in

$a \times 1 = a$
$1 \times a = a$

Multiplicative inverse - $\frac{1}{a} \cdot ax = b \cdot \frac{1}{a}$ what you multiply by to get an answer of 1

$2 \cdot \frac{1}{2} = 1$
$\frac{1}{2} \cdot 2 = 1$

Identity matrix [I]- $[A][I] = [A]$

Square matrix $[I][A] = [A]$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Inverse Matrix $[A]^{-1}$- $[A][A]^{-1} = [I]$

$[A]^{-1}[A] = [I]$

Ex. A $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [I]$

$$\begin{bmatrix} 2a+1c & 2b+1d \\ 4a+3c & 4b+3d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$2\times$ $2a+c=2$

$4a+3c=4$

$-\ 4a+2c=4$

$c=0$

$2a+0=2$

$\frac{2a}{2}=\frac{2}{2}$

$a=1$

$2\times$ $2b+d=1$

$4b+3d=3$

$-\ 4b+2d=2$

$d=1$

Identity Matrix

only contains 0 and 1

$[A][I] = [A]$

$[I][A] = [A]$

Square matrix

$[A] = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ $[A]^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $[A][A]^{-1} = [I]$

$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 3/2 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = [A]^{-1}$

$\begin{bmatrix} 2a+c & 2b+d \\ 4a+3c & 4b+3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2x$ $2a + c = 1$
$4a + 3c = 0$
$-\; 4a + 2c = 2$

$c = -2$

$2a - 2 = 1$
$+2 \quad +2$
$\frac{2a}{2} = \frac{3}{2}$
$a = \frac{3}{2}$

$3x$ $2b + d = 0$
$4b + 3d = 1$
$-\; 6b + 3d = 0$

$\frac{-2b}{-2} = \frac{1}{-2}$
$b = -\frac{1}{2}$

$2\left(-\frac{1}{2}\right) + d = 0$
$-1 + d = 0$
$+1 \quad +1$
$d = 1$

Solving a system using an inverse matrix

$$[A][X]=[B]$$

$$[A]^{-1}[A][X]=[B][A]^{-1}$$

$$[I][X]=[B][A]^{-1}$$

$$[X]=[B][A]^{-1}$$

Ex. B p. 329 4/25/12

$2x + 3y = 7$

$x = 6 - 4y$

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$[X] = [B][A]^{-1}$

$[A]$

$[A]^{-1}$

$[A][A]^{-1} = [I]$

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$2a + 3c = 1$ $2b + 3d = 0$

$a + 4c = 0$ $b + 4d = 1$

$2a + 8c = 0$ $2b + 8d = 2$

$-\ 2a + 3c = 1$ $-\ 2b + 3d = 0$

$5c = -1$ $5d = 2$

$c = -\frac{1}{5}$ $d = \frac{2}{5}$

$a + 4\left(-\frac{1}{5}\right) = 0$

$a + -\frac{4}{5} = 0$

$a = \frac{4}{5}$

$b + 4\left(\frac{2}{5}\right) = 1$

$b + \frac{8}{5} = 1$

$-\frac{8}{5}$ $-\frac{8}{5}$

$b = -\frac{3}{5}$

$$\begin{bmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{bmatrix}$$

1

$$[B] = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \qquad [A]^{-1} = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

$$2 \times 1 \qquad\qquad 2 \times 2$$

$$2 \times 2 \qquad 2 \times 1$$

$$\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 7 \\ 6 \end{bmatrix} = \begin{bmatrix} \left(\frac{4}{5}\right)7 - \left(\frac{3}{5}\right)6 \\ -\left(\frac{1}{5}\right)7 + \left(\frac{2}{5}\right)6 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ y \end{bmatrix}$$

Ex. C p. 330

Candy = C
Drink = D
Popcorn = P

$$[A]\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} C \\ D \\ P \end{bmatrix} = \begin{bmatrix} 11.85 \\ 9 \\ 12.35 \end{bmatrix}[B]$$

$$\begin{matrix}\$ \\ \$ \\ \$\end{matrix}\begin{bmatrix} C \\ D \\ P \end{bmatrix} = \begin{bmatrix} 2.15 \\ 2.05 \\ 2.75 \end{bmatrix}$$

p. 331-332 6.4
1-6

6.5 Systems of Linear Inequalities p. 336 4/30/12

sides of the expression are not equal

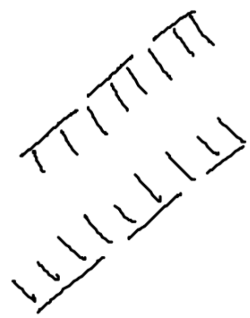
$< \quad > \quad \leq \quad \geq$

$<$ and $>$: dashed — the line is NOT included

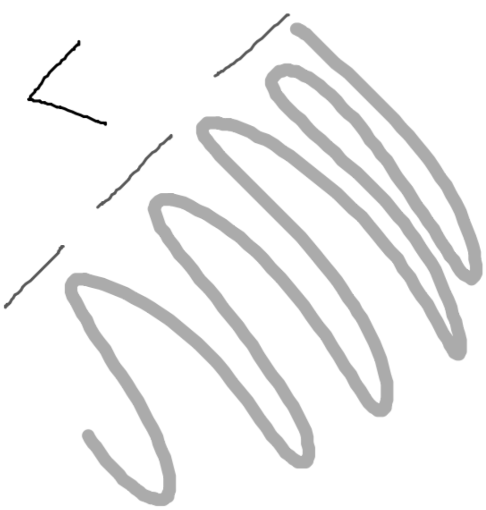
$\leq$ and $\geq$: solid — the line is included

$< \quad \leq$

$> \quad \geq$



$<$

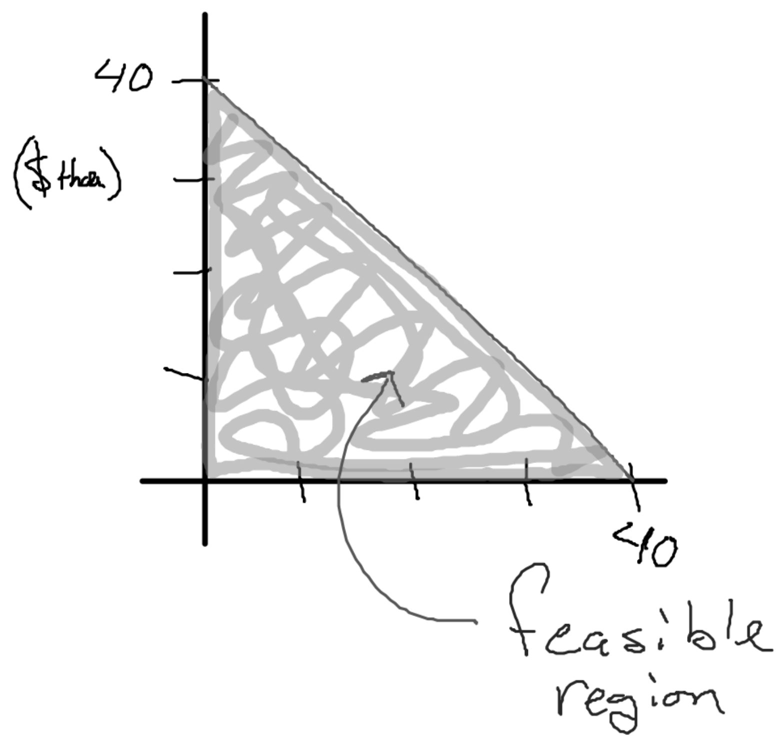


$\leq$

Inv. 40,000 $x + y = 40000$ 5/02/12

x Stocks $x \le 40,000$ } Constraints

y Bonds $y \le 40000$



Ex. x math y chemistry

5/02/12

- $x + y \leq 3$
- $y \geq \frac{1}{2}$
- $x > y$

$\frac{5-2x}{2}$ 2.5−1x

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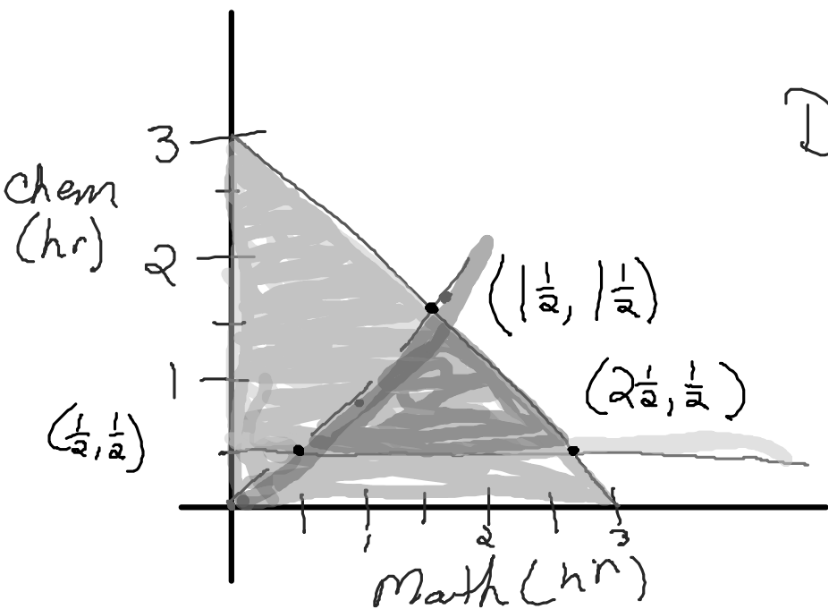
#1–3, 5



Divide or multiply by a negative, flip the symbol

$< \leftrightarrow >$

Maximize or minimize the value of a function.

Inv.

	Amount per unglazed birdbath u (x)	Amount per glazed birdbath b (y)	Constraining value
Wheel hours	.5 h	1	≤ 8 h
Kiln hours	3 h	18	≤ 60 h
Profit	$10	$40 y	Maximize

$.5u + 1b \le 8$

$3u + 18b \le 60$

≥ 6

$1b \le 8 - .5u$

$\frac{18b}{18} \le 60 - 3u$

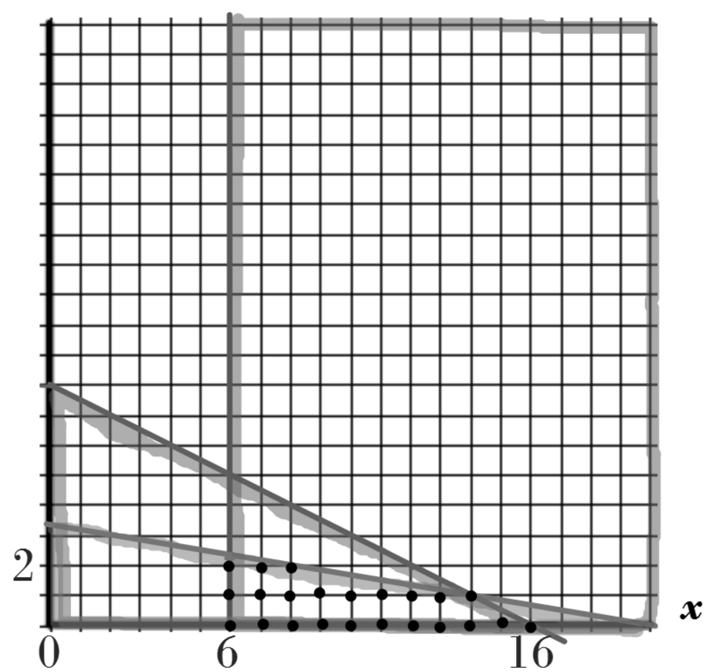
$b \le 3\frac{1}{3} - \frac{1}{6}u$

$p = 10u + 40b$

$10(16) + 40(0) = \$160$

→ $10(14) + 40(1) = \$180$

$10(8) + 40(2) = \$160$



Ex. 5/04/12

		Serving	Calories	Fat	Protein	Iron (percent of daily recommended value)	Cost($)
(x) g	Graham crackers	1 cracker	60	2 g	2 g	6%	0.06
(y) b	Blueberry yogurt	4.5 oz	130	2 g	5 g	1%	0.30
			≤ 700	≤ 20	≥ 17	$\geq 30\%$	minimize

C $60g + 130b \leq 700$

F $2g + 2b \leq 20$

P $2g + 5b \geq 17$

I $6g + 1b \geq 30$

$b \leq \frac{700 - 60g}{130}$

$b \leq \frac{20 - 2g}{2}$ $10 - g$

$b \geq \frac{17 - 2g}{5}$

$b \geq 30 - 6g$

P. 347

#1-3

5/07/12

$C)\ b \le \frac{70}{13} - \frac{6}{13}g$

$F)\ b \le 10 - g$

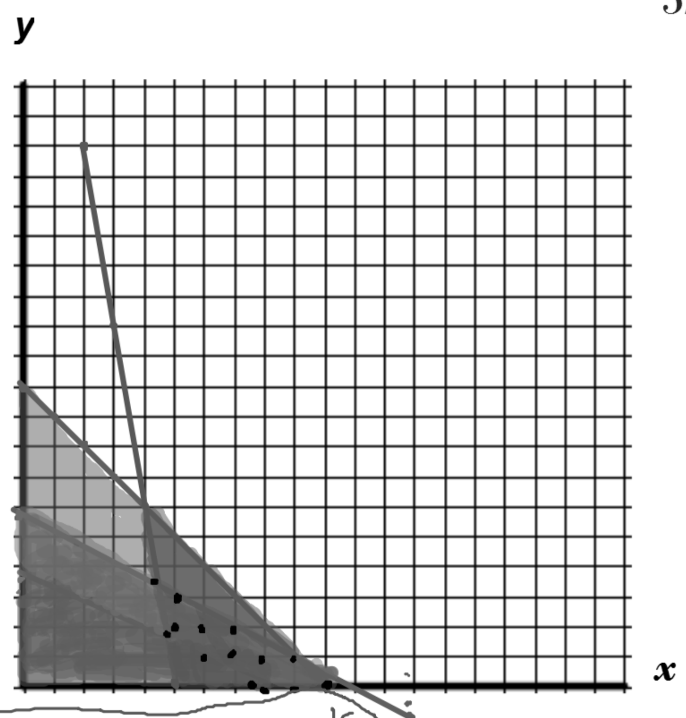
$P)\ b \ge \frac{17}{5} - \frac{2}{5}g$

$I)\ b \ge 30 - 6g$

5,2 6,1 7,1 8,1
5,3 6,2 7,2 9,1

(9,0)

(9,0) \$ = 0.54



Homework
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#1-3

Chapter 6 Review pp. 351-352 5/11/12

1, 2 a-c, 3-5, 7

- 2nd Matrix Edit Rows x Columns
  enter data
  2nd Quit
  Repeat for answer matrix or additional matrices

- for Inverse matrix answer

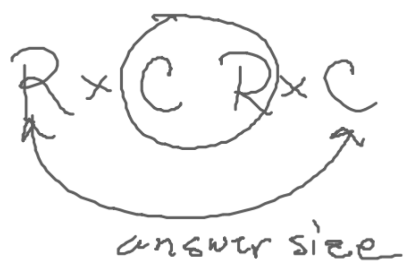
  2nd Matrix Name
  [A]
  X^{-1}
  2nd Matrix Name
  [B]
  enter

answer matrix

$$\left[\begin{array}{cc|c} \overset{x}{1} & \overset{y}{0} & \overset{=}{a} \\ 0 & 1 & b \end{array}\right]$$

$<$ $>$

$\leq$ $\geq$



$$x + y = a$$
$$x + y = b$$